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Simulation of the Transient Heating in an Unsymmetrical, Coated, Hot-Strip Sensor with a Self-Adaptive Finite-Element Method (SAFEM)[1]

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The transient heating in an unsymmetrical, coated, hot-strip sensor was simulated with a self-adaptive finite-element method (SAFEM). The first tests of this model show that it can be used to determine, with a small error, the thermal conductivity of liquids, from the transient temperature rise in the hot strip, which is deposited in a substrate and coated by an alumina spray.

KEY WORDS: molten materials; self-adaptive finite-element method; thermal conductivity; transient hot-strip.

1. INTRODUCTION

Measurement of the thermal conductivity of molten materials is very difficult mainly because the mathematical modeling of the heat transfer processes at high temperatures with several different media present is far from being solved. However, the scatter of the experimental data presented by different authors using several methods is so large that any scientific or technological application is strongly limited without serious approximations.

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The development of a new instrument for the measurement of the thermal conductivity of molten salts, metals, and semiconductors implies the design of a sensor for the measurement of temperature profiles in the melt, apart from the necessary electronic equipment for the data acquisition and processing, furnaces, and gas/vacuum manifolds. This paper describes the development of a powerful algorithm, based on the self-adaptive finite-element method (SAFEM) to analyze the temperature profile in a sensor previously constructed and characterized [1, 2]. This characterization showed that the platinum thin film is homogeneous, with a stable resistance up to 1100°C, and a temperature coefficient of resistance equivalent to the best platinum resistance thermometers for high temperature measurements. Therefore, a good bond between the thin film and the substrate has been obtained which indicates the absence of thermal barriers for heat conduction usually present in the production of electronic components not using physical vapor deposition.

These conclusions simplify drastically the general equation for heat conduction, as no additional terms resulting from contact heat resistances have to be included.

2. THEORY

A planar, electrically conducting (metallic) element is mounted within an insulating substrate material which is surrounded by a material whose thermal properties have to be determined. From an initial state of equilibrium, Ohmic dissipation within the metallic strip q begins at time $t=0$, and results in a temperature rise on the strip, and a conductive thermal wave spreads out from it through the substrate into the testing material. The temperature history of the metallic strip, as indicated by its change of electrical resistance, is determined, in part, by its own thermal conductivity and thermal diffusivity, in part by the properties of the substrate, and in part by the material thermal conductivity and thermal diffusivity. The working equation used in transient techniques to obtain the thermal conductivity value of a viscous, isotropic, and incompressible fluid with temperature-independent properties is the energy conservation equation, which under some conditions, can be transformed to

$$\rho C_P \frac{DT}{Dt} = \nabla \cdot (\lambda \nabla T) + Q \tag{1}$$

The values of the material properties are piecewise constant. This equation has to be applied to three distinct regions: to the strip, to the substrate, and to the material. Until now it has not been possible to solve this equation analytically. The complexity of the geometry of the sensor developed in Lisbon results in a three-dimensional problem. In a first attempt, the end effects are neglected along the longitudinal dimension of the sensor. A sequence of linear heat transfer equations in two dimensions is solved for the cross section of the metal strip to obtain the thermal conductivity parameter λ of the surrounding material.

In Eq. (1), Q is zero for all materials except for the strip, where it is a nonzero constant $Q_{strip} = q/(abl)$. Because the experiment is carried out in short periods of time, we assume that no convection can occur.

Figure 1 shows a schematic of the metal strip (material 1), the alumina substrate (material 3), the alumina coating (material 2), and the molten material (material 4).

In the theoretical treatment it is assumed that the system is infinitely long in the z-direction, perpendicular to the page ($l_{\text{theoretical length}} \to \infty$). Therefore, T depends only on x and y, and $\partial T/\partial z = 0$. This allows us to study only a two-dimensional problem. All the other relevant geometrical aspects of the system are defined in Fig. 1.

It is assumed that, from an initial equilibrium state in which $\Delta T = 0$ or $T = T_0$ everywhere, heat is generated in the strip at a rate Q. In addition, for each time step, the condition of thermal equilibrium in each volume element is imposed because the time scale of the experiment (20 ms to 1 s)

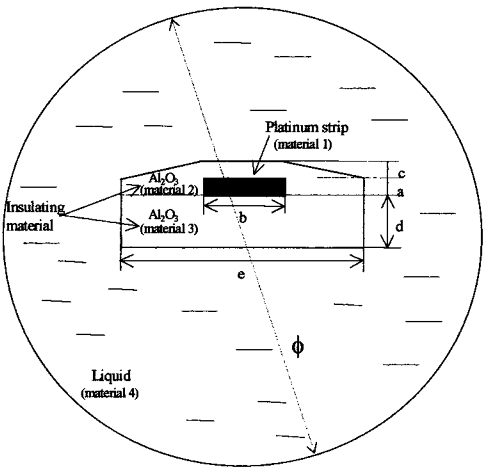

Fig. 1. Schematic of the hot-strip sensor (not to scale); (a) active metal film thickness; (b) active metal film width; (c) insulating coating thickness; (d) substrate thickness; (e) substrate width; ϕ, crucible diameter.

is large enough to avoid short transient instabilities in the thin film. For boundary conditions we assume that $\Delta T = 0$ or $T = T_0$ on the boundary of the square domain. We place this artificial boundary far enough from the sensor so that no significant part of the heat generated at the strip reaches the boundary during the time of the experiment.

We use a variational formulation that implies the transmission conditions. For two different materials i and j with temperature distributions T_i and T_j separated by a surface Γ, we always have $T_i = T_j$ on Γ and $\lambda_i(\partial T_i/\partial n) = \lambda_j(\partial T_j/\partial n)$ across Γ, for $t > 0$, where $\partial T_i/\partial n$ and $\partial T_j/\partial n$ denote the normal derivatives on Γ.

For the geometry being used we can profit from the vertical symmetry axis at the middle of the system and simulate the evolution of temperature only in the right half of the domain. On the symmetry axis we impose a homogeneous Neumann boundary condition, $\partial T/\partial n = 0$.

The heat equation can be solved analytically for simple geometries. However, when the model is more complex, we only expect to obtain numerical approximate solutions. In our case the transient problem, Eq. (1), is reduced to a sequence of stationary elliptic problems. Each of these problems is solved using a finite-element method. Finite-element methods are based on an integral formulation of the equations and are very well suited to problems for complex geometries or for the case of different materials.

3. ADAPTIVE NUMERICAL ALGORITHM

We are interested in solving the linear heat transfer Eq. (1) in two space dimensions in the absence of fluid movement:

$$\rho C_P \frac{\partial T}{\partial t} = \nabla \cdot (\lambda \nabla T) + Q_{\text{strip}} \tag{2}$$

$$\frac{\partial T}{\partial n}(0, y, t) = 0, \qquad T = 0 \quad \text{elsewhere on the boundary} \tag{3}$$

$$T(x, y, 0) = 0 \tag{4}$$

for the different thermal conductivities λ. Due to the strongly localized source Q and the different properties of the involved materials, we initially observe steep gradients of the temperature profiles, which decrease with time. To ensure good resolution in space and efficient time integration for such a situation, a method with automatic control of spatial and temporal discretization is recommended. We use the programming package KARDOS [3–5] to provide these desired features.

Starting with a guess, we improve the parameter iteratively until the computational result is a good approximation of the measured data. For a fixed value of λ the computation of the heat transfer in the sensor and the surrounding liquid is based on the self-adaptive finite-element code KARDOS developed at the Konrad-Zuse-Zentrum in Berlin. An implicit time integrator of Rosenbrock type is coupled with a multi-level approach in space. Local *a posteriori* estimates based on higher order solutions are computed to assess the spatial discretization. The resulting elliptic equations are solved by an adaptive multi-level finite-element method. Because of the very different sizes of the involved elements (sensor components, liquid), it is necessary to provide an appropriate initial grid in order to guarantee numerical stability during the adaptive mesh refinement. These estimations are used to decide where further refinement is necessary and where the mesh can be coarsened without losing accuracy. Such adaptive strategies, including step size control in time, spatial refinement, or coarsening combined with multi-level solvers, allow computing efficiently an accurate solution. In Fig. 2 we present a typical adaptive grid with the corresponding picture of the solution.

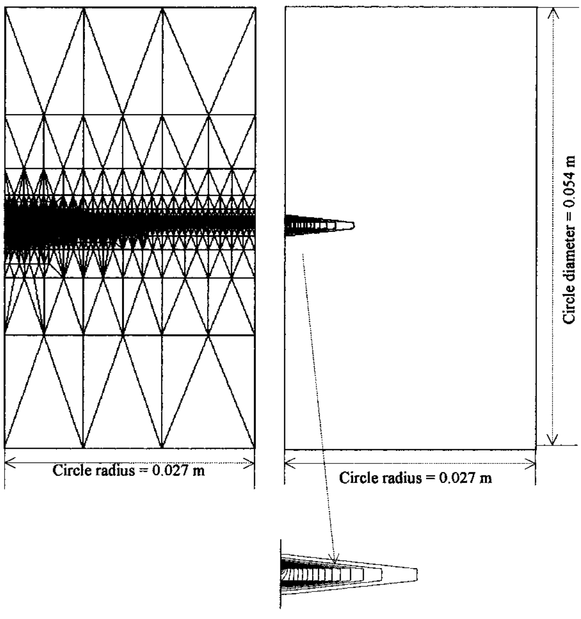

Fig. 2. Adaptive grid and isotherms of the corresponding solution at time $t = 1.0$.

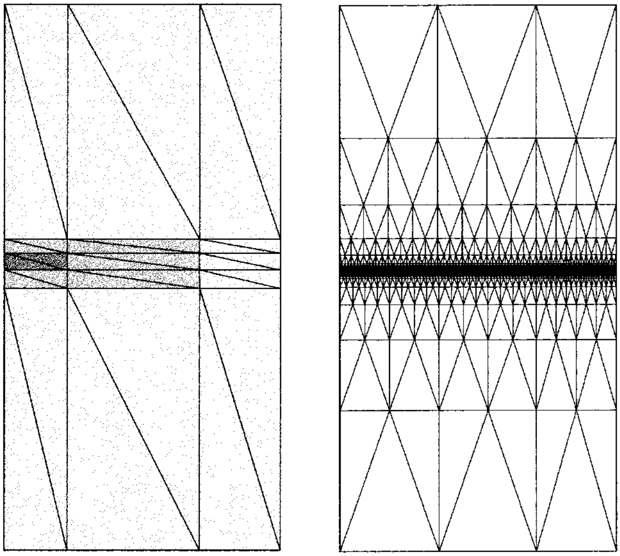

Fig. 3. Left: Schematic of strip, isolating material, and liquid. Right: Initial grid.

A special difficulty arises from the multi-scaling character of the configuration: the tremendous differences in the sizes of the involved elements (sensor components, liquid). Taking a very coarse triangulation (see Fig. 3, left) of these elements as an initial grid for our computations would result in large discretization errors and lead to instability during the time integration. No further refinement of such a bad grid (because of the obtuse angles) can remedy the instability. Consequently, we have to provide a reasonable initial grid (see Fig. 3, right) to start our adaptive method, i.e., we have to avoid obtuse angles and to generate a smooth transition from fine to coarse mesh size [6].

4. RESULTS

From measurements we know the average temperatures in the platinum strip as a function of time [7]. In a first numerical attempt we try to simulate the heat conduction. Starting with a guess for the parameter λ, we compute the temperature up to time $t = 1.0$. If the error between measurement and computed data is too large, we choose a new value for λ and repeat the calculation. Here we present our results for the determination of the thermal conductivity of water (see Figs. 4 and 5).

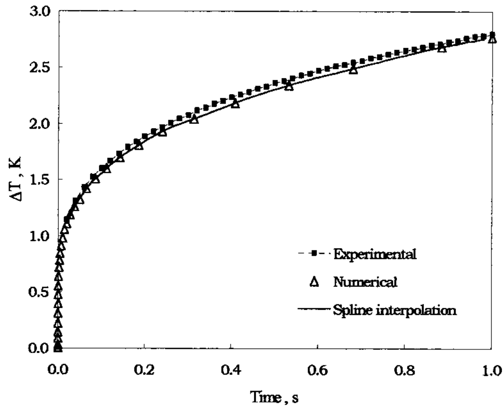

Fig. 4. Simulation for water at 25°C; computation at 12 V, 37.5 $W \cdot m^{-1}$, $\lambda_1 = 72.0$, $\lambda_2 = 25.5$, $\lambda_3 = 32.3$, in $W \cdot m^{-1} \cdot K^{-1}$ and $c = 4.7\ \mu m$.

Figure 4 shows the results obtained for the sensor described in [1] and [2], without considering the very thin titanium layer between the substrate and the platinum thin film. The agreement is quite satisfactory, and it results in a water thermal conductivity of 0.606 $W \cdot m^{-}1 \cdot K^{-1}$ at 25°C, a value within 0.1% of the recommended thermal conductivity for this liquid at this temperature (0.6065 ± 0.0036 $W \cdot m^{-}1 \cdot K^{-1}$) [8]. Figure 5 shows the deviations between the experimental points and a spline interpolation of the numerical points. The deviations are almost systematic, although with a slightly different slope. This might be caused by the fact that the numerical calculation did not consider the power necessary to heat

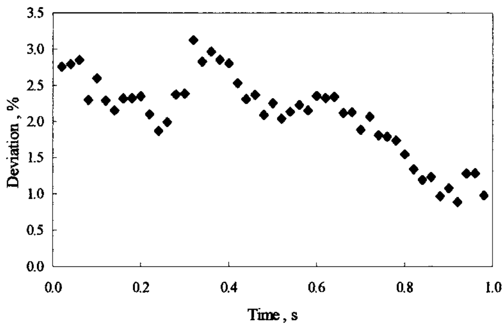

Fig. 5. Deviations between the experimental points and the spline interpolation of the numerical points.

the thin layer of titanium ($\approx$9.8 nm). Preliminary numerical calculations with mercury, NaCl aqueous solutions, and toluene show the same trend.

5. CONCLUSIONS

A very powerful self-adaptive finite-element method (SAFEM) was applied to solve numerically the heat transfer in a resistive temperature sensor to be used in the measurement of the thermal conductivity of molten materials at high temperatures. The results so far obtained at room temperature look very promising and can form the basis for the application of these metal thin-film sensors to measure the thermal conductivity of liquids at temperatures up to 1500 K.

The program KARDOS can also be exploited for the simulation of more complicated models, e.g., using heat conduction in three space dimensions or nonlinear dependences, like the contribution of radiation to heat transport, which can be important at high temperatures. We hope to present the solution for the three-dimension sensor in the near future.

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